A Conjecture

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Let $0 < q < 1, \quad k$ large integer. Let the matrix P defined by

$$P(u,v) := \binom{k}{v}(1-q^u)^v q^{u(k-v)}, \quad u = 1..k, v = 1..k.$$

Conjecture:
The dominant eigenvalue of P is given by $1-\theta$,

$$\theta = q^{k^2}2^k + C_1 q^{k^2} + C_2 q^{k^2}/2^k + \dots$$

with corresponding right eigenvector

$$(1-\eta_1, \dots, 1-\eta_k), \quad \eta_u \sim q^{uk},$$

and corresponding left eigenvector

$$(\varepsilon_1, \dots, \varepsilon_{k-1}, 1-\varepsilon), \quad \varepsilon \sim kq^k, \quad \varepsilon_v \sim \binom{k}{v} q^{k(k-v)}.$$

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